

# $Q^2$ evolution of $^3\text{He}$ spin structure functions moments

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Received: 11 October 2004 / Published Online: 8 February 2005  
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**Abstract.** Sum rules offer the opportunity to test our understanding of the structure of a system on general grounds. In this paper we investigate the  $^3\text{He}$  nucleus using the Gerasimov–Drell–Hearn and Burkhardt–Cottingham sum rules. We have previously used the polarized  $^3\text{He}$  target measurements to extract the neutron spin structure, our goal here is to investigate rather the  $^3\text{He}$  nucleus. We have determined for the first time the  $Q^2$  evolution of  $\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2) dx$  and  $\Gamma_2(Q^2) = \int_0^1 g_2(x, Q^2) dx$  for  $^3\text{He}$  in the range  $0.1 \text{ GeV}^2 \leq Q^2 \leq 0.9 \text{ GeV}^2$  with good precision.  $\Gamma_1(Q^2)$  displays a smooth variation from high to low  $Q^2$ . However, we do not see yet its expected turnover toward the real photon limit value. On the other hand the Burkhardt–Cottingham sum rule holds at all measured  $Q^2$ s within uncertainties due mainly to cancellations between the elastic, quasielastic and the  $\Delta(1232)$  resonance contributions.

## 1 Introduction

In the past few years a series of experiments have been performed around the world to investigate the spin structure of the proton, deuteron and  $^3\text{He}$  using real photons or low momentum transfer ( $Q^2 \leq 2\text{GeV}^2$ ) virtual photons. The experiments performed on the light nuclear targets were aimed at extracting the neutron spin properties. In these extractions only data above the single pion production threshold were used. However, during most of these experiments the spin structure functions of nuclei are typically measured including the quasielastic region of excitation. This region is important in the evaluation of the sum rules for nuclei.

It is interesting to note that the spin sum rules used in the investigation of the nucleon have their analog in the case of nuclei. However, the contributions of the excitation spectrum by which these sum rules are verified are different in nuclei compared to the nucleon. At low momentum transfer the coherent behavior of the constituents is likely to emerge as an important ingredient in understanding the structure of a complex system as a nucleus. The coherence of quark and gluon interactions manifest itself ultimately in the binding of protons and neutrons inside the nucleus. Thus both the quasielastic and the resonance regions play an important role in contributing to the sum rules of nuclei.

In this paper I shall focus on a measurement carried out by the JLab E94010 collaboration using a polarized  $^3\text{He}$  target. I shall present results of the quasielastic region and quantities described by the generalized Gerasimov–Drell–Hearn sum and the Burkhardt–Cottingham sum and study their  $Q^2$  evolution.

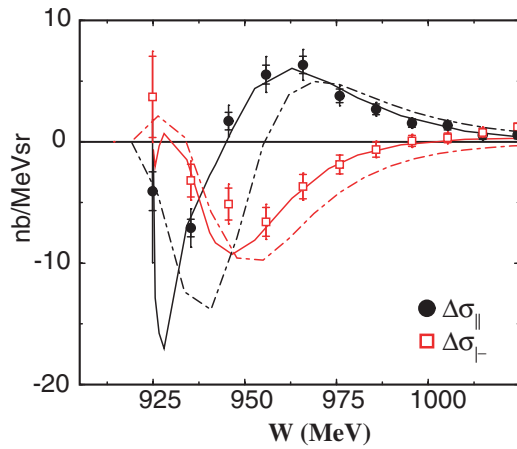
## 2 Experiment E94-010

We measured the inclusive scattering of longitudinally polarized electrons off a polarized  $^3\text{He}$  target at the Thomas Jefferson National Accelerator Facility (JLab) in Hall A. Data were collected at six incident beam energies: 5.058, 4.239, 3.382, 2.581, 1.718, and 0.862 GeV, all at a nominal scattering angle of  $15.5^\circ$ . The measurements covered values of the invariant mass  $W$  from the threshold of quasielastic scattering (not discussed in previous references), through the resonance region and continuum. Data were taken for both longitudinal and transverse target polarization orientations. Both spin asymmetries and absolute cross sections of  $^3\text{He}$  were measured. In this experiment we achieved a very high “polarized” luminosity ( $\mathcal{L} 10^{36} \text{ (cm}^2 \cdot \text{s)}^{-1}$ ). Results from this experiment were published starting from an invariant mass corresponding to the pion production threshold off the nucleon and focused on the spin structure of the neutron extracted from a  $^3\text{He}$  target. Here and for the first time we focus our attention on the  $^3\text{He}$  nucleus itself and investigate its spin structure. In this case the contribution of the quasielastic is critical to evaluate quantities like the generalized Gerasimov–Drell–Hearn sum and the Burkhardt–Cottingham sum. Further experimental details can be found in [1,2].

### 2.1 Quasielastic scattering off $^3\text{He}$

While the excitation region beyond the quasielastic peak in JLab E94010 was used earlier to extract the spin structure of the neutron, no quasielastic data were analyzed at that time. Nevertheless, this region is interesting in its own right and is required before any moments of  $^3\text{He}$  structure functions can be evaluated. Therefore we have

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**Fig. 1.** Preliminary data of polarized cross section differences in the quasielastic region at incident electron energy  $E_0 = 0.9$  GeV. The *solid circles* are for the case where the beam and target polarizations are longitudinal. The *open squares* are results in the case where the beam and target polarization are perpendicular. The *inner error bars* include statistical uncertainties only while the *outer ones* combine the statistical and the systematic uncertainties in quadrature. The *dot-dashed lines* are plane wave impulse approximation calculations [3]. The *solid lines* represent a full three-body Faddeev calculations [4]

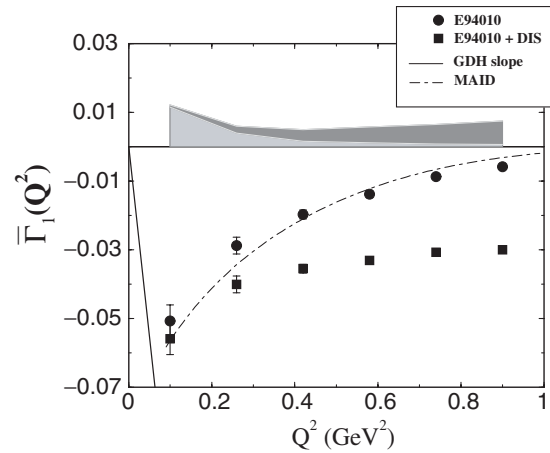
performed a measurement of the spin dependent cross sections in this region and subsequently determined the spin structure functions  $g_1(x, Q^2)$  and  $g_2(x, Q^2)$  of  $^3\text{He}$ .

In a nucleon-meson description of the  $^3\text{He}$  nucleus the quasielastic region is dominated by the knock-out of a moving nucleon in the nucleus. At low momentum transfer this region is sensitive to nucleon-nucleon long-range correlations. Furthermore, meson-exchange currents play an important role in a complete description of this region, especially when the momentum transfer becomes large. Full three body Faddeev calculations exist and have already been used to extract the neutron electromagnetic properties at low momentum transfer [4]. We first compare in Fig. 1 the results obtained at the lowest incident electron beam energy  $E_0 = 0.9$  GeV. It is very encouraging to see the good agreement for both the transverse and longitudinal spin dependent cross sections differences. These results point again at the importance of treating the ground state and scattering states of  $^3\text{He}$  in a consistent manner in order to include nucleon-nucleon correlations and final state interactions properly. We have also verified that the unpolarized cross section agrees very well with results from Bates and Saclay on  $^3\text{He}$  [2].

## 2.2 The generalized Gerasimov–Drell–Hearn sum rule in $^3\text{He}$

In its original derivation the Gerasimov–Drell–Hearn [5] sum rule connects the energy-weighted integral of the helicity dependent photoabsorption cross sections difference

$$\int_{\nu_0}^{\infty} \left[ \sigma^{\rightleftharpoons}(\nu) - \sigma^{\leftarrow}(\nu) \right] \frac{d\nu}{\nu} = -\frac{4\pi^2 J \alpha}{M^2} \kappa \quad (1)$$



**Fig. 2.** Preliminary results of  $\bar{T}_1(Q^2)$  for  $^3\text{He}$ . See text for details

to the anomalous magnetic moment  $\kappa$  of a given system with spin  $J$  and mass  $M$ . Here  $\nu$  is the energy of the photon and  $\nu_0$  is the threshold energy of photoabsorption. Thus  $\nu_0$  corresponds to the pion production energy threshold when we consider a nucleon target while it is the energy transfer required for the two-body breakup in a nucleus like the deuteron or  $^3\text{He}$ . The sum rule prediction is  $-234 \mu\text{b}$  for a neutron target and  $-496 \mu\text{b}$  for  $^3\text{He}$ . Part of the origin of such a difference lies mainly in the contribution of the threshold region as pointed out by Arenhövel et al in an extensive theoretical study in the case of the deuteron [8].

This sum rule has been generalized [6] to include the case of virtual photoabsorption, namely the case of inelastic lepton scattering at arbitrary virtuality  $Q^2$  (scale). Various generalizations of the above expression have been proposed in the literature [7] but they all converge to the same integral at large values of  $Q^2$ . One example is given by

$$I_{GDH}(Q^2) = \int_{\nu_0}^{\infty} \left[ \sigma^{\rightleftharpoons}(\nu, Q^2) - \sigma^{\leftarrow}(\nu, Q^2) \right] \frac{d\nu}{\nu} \quad (2)$$

$$\rightarrow \frac{16\pi^2 \alpha}{Q^2} \bar{T}_1(Q^2) \quad (3)$$

where  $\sigma^{\rightleftharpoons}(\nu, Q^2)$  ( $\sigma^{\leftarrow}(\nu, Q^2)$ ) describes the photoabsorption cross section of virtual photons of helicity parallel (anti-parallel) to the target polarization. In  $\bar{T}_1(Q^2)$  the integration stops at the threshold of inelasticity  $x_{thr}$ , and thus does not include the elastic contribution. For a nuclear system like  $^3\text{He}$ ,  $x_{thr}$  corresponds to the two-body break-up excitation energy. In this case the integral over the excitation spectrum will also include a significant contribution from the quasielastic region (see Fig. 1).

In order to investigate the  $Q^2$  evolution of the generalized GDH sum rule we choose to use  $\bar{T}_1(Q^2)$  across the full range of  $Q^2$ . Its limit at  $Q^2 \rightarrow 0$  is given by

$$\bar{T}_1(Q^2) = -\frac{Q^2}{8M^2} \kappa^2 + O\left(\frac{Q^4}{M^4}\right), \quad (4)$$

We present in Fig. 2 the results of  $\bar{\Gamma}_1(Q^2)$  for  $^3\text{He}$ . The filled circles labeled E94-010 represent our measured data with their statistical uncertainties for the part of the integral from the two-body breakup of  $^3\text{He}$  up to an invariant mass  $W = 2$  GeV. The systematic error is represented by the light band. The solid squares are obtained after adding an estimate of the inelastic contribution from  $W = 2$  GeV to  $W = \sqrt{1000}$  GeV using the Thomas and Bianchi parametrization [10]. The dark band is an estimate of the total error obtained after including the latter contribution. The GDH result corresponds to the slope at  $Q^2 = 0$  depicted by the solid line. The dot-dashed line is an estimation which combines a PWIA evaluation of the quasielastic and resonance regions response. In the resonance region the phenomenological MAID model [7] is used to evaluate the protons and neutron response functions.

We note that  $\bar{\Gamma}_1$  has a smooth variation from high to low  $Q^2$ . It does not yet show the expected turnover towards zero with the GDH prediction slope. We hope that in the future these data along with the expected data from [19] will provide the impetus for the calculation of the nuclear spin response at very low  $Q^2$  within chiral perturbation theory.

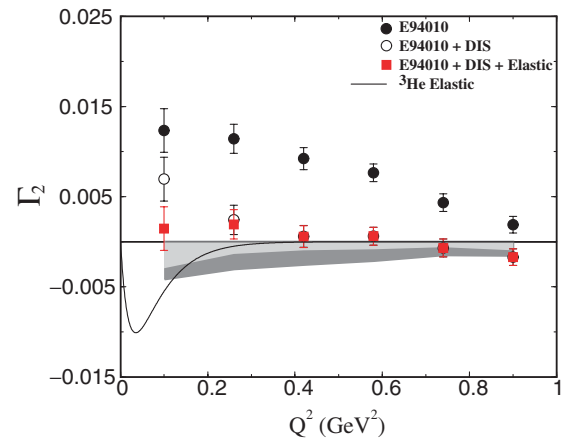
### 2.3 The Burkhardt–Cottingham sum rule in $^3\text{He}$

The  $g_2$  structure function is predicted to obey the Burkhardt–Cottingham (BC) sum rule

$$\Gamma_2(Q^2) \int_0^1 g_2(x, Q^2) dx = 0 \quad (5)$$

which was derived from the dispersion relation and the asymptotic behavior of the corresponding Compton amplitude [9] for the case of the nucleon. This sum rule is also expected to be valid at all  $Q^2$  for the nucleon and spin 1/2 nuclei. It is a super-convergence relation based on Regge asymptotics as discussed in the review paper by Jaffe [11]. Many scenarios which could invalidate this sum rule in the case of the nucleon have been discussed in the literature [12, 13, 14]. However, this sum rule was confirmed in perturbative QCD at order  $\alpha_s$  with a  $g_2(x, Q^2)$  structure function for a quark target [16]. Surprisingly the first precision measurement of  $g_2$  at SLAC [15] at  $Q^2 = 5$  GeV<sup>2</sup> but within a limited range of  $x$  has revealed a violation of this sum rule on the proton at the level of three standard deviations. In contrast, the neutron sum rule is poorly measured but consistent with zero at the one standard deviation. Here we are interested in testing the BC sum rule in  $^3\text{He}$ .

In Fig. 3 we present the results of the BC sum. The solid circles represent the results of the integration of  $g_2$  in our measured range which includes the quasielastic and resonances regions up to  $W = 2$  GeV. An estimation of the unmeasured inelastic region is performed using the leading twist Wandzura-Wilczek relation [17] between  $g_1$  and  $g_2$  and added (open circles). The total integral (solid squares) includes the contribution of elastic scattering off



**Fig. 3.** Preliminary results of  $\Gamma_2(Q^2)$  for  $^3\text{He}$ . See text for details

$^3\text{He}$  (solid curve) evaluated using the form factors of [18]. The grey band corresponds to the systematic uncertainty of the measurements while the dark band represents total error after adding quadratically the systematic error due to the inelastic unmeasured contribution.

It is interesting to note that the BC sum rule is verified with uncertainties at all momentum transfers measured in this experiment. This is a non-trivial result since in each region of  $Q^2$  the cancellations of contributions to the integral arise from different processes.

### 2.4 Conclusion

We have measured the spin structure functions  $g_1$  and  $g_2$  of  $^3\text{He}$  in the quasielastic and resonance regions in the range  $0.1 \leq Q^2 \leq 0.9$  GeV<sup>2</sup>. In the quasielastic region the data agree with a full Faddeev calculation. We have determined the GDH and BC sums in  $^3\text{He}$ . The variation of  $\bar{\Gamma}_1$  is smooth from large to small  $Q^2$ . Down to  $Q^2 = 0.1$  GeV<sup>2</sup> we do not see yet its expected turnover towards zero, with a slope proportional to the anomalous magnetic moment of  $^3\text{He}$ . A new experiment under analysis should provide this information soon [19]. The BC sum rule seem to hold within uncertainties in the measured  $Q^2$  range due to subtle cancellations among the elastic, quasielastic, resonance and deep inelastic contributions depending on the value of  $Q^2$ .

*Acknowledgements.* I thank the organizers for their invitation to a productive meeting in a beautiful setting. This work is supported in part with funds provided to the Nuclear and Particle Group at Temple University by the U.S. Department of Energy (DOE) under contract number DE-FG-02-94ER-40844.

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